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A 2-D photonic crystal based source of polarization entangled photon pairs with high nonlinear conversion efficiency and without walk-off compensation

Li Xiao *, Yin Wang, Wei Zhang, Yidong Huang, Jiangde Peng

Department of Electronic Engineering, Tsinghua University, Beijing 100084, China

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Abstract

Spontaneous parametric down-conversion in 2-dimensional photonic crystal made of semiconductor material with large nonlinear susceptibility is proposed to generate entangled photon pairs with high nonlinear conversion efficiency. In particular, the walk-off between the down-converted photons with orthogonal polarization states can be minimized through appropriate structure parameter design, leading to elimination of compensation measures to mitigate labeling effect on polarization entangled photon pairs.

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Entangled photon pairs are important in fundamental research in quantum mechanics and physical realization of quantum information technologies [1–4]. Spontaneous parametric down-conversion (SPDC) in nonlinear crystals, such as non-collinear type II phase-matching β-barium-borate (BBO), is commonly utilized to generate polarization entangled photon pairs [5]. However, besides low nonlinear conversion efficiency, "walk-off" caused by natural birefringence of crystals [5] is also a serious issue which leads to the separation of down-converted photons with orthogonal polarization states in both space and arrival time at the collecting point. Such walk-off can be compensated for by incorporating additional crystals in each path. In order to increase the nonlinear conversion efficiency, a source based on 1-dimensional (1-D) photonic crystal (PC) of $Al_{0.4}Ga_{0.6}As$/air has been proposed to generate polarization entangled photon pairs at 1500 nm [6]. The nonlinear conversion efficiency is improved due to the large quadratic nonlinear susceptibility $\chi^{(2)}$ of AlGaAs, while phase-matching for SPDC can be achieved by using a combination of form birefringence and phase velocity dispersion in a periodic structure [7]. The quantum correlation property of down-converted photon pairs in the PC was proved by quantum model [8,9]. SPDC in PC waveguide was also proposed [10]. However, the walk-off between photons with orthogonal polarization states in entangled photon pairs generated in the present PC structures still exists as in traditional nonlinear crystals.

In this letter, we theoretically investigate SPDC in a 2-dimensional (2-D) PC composed of AlGaAs/air to generate polarization entangled photon pairs. The nonlinear interaction among Bloch waves in a 2-D PC is analyzed by the method given in Ref. [6]. Since the structure parameters of 2-D PC related to the group velocity dispersion is more flexible to adjust than those of 1-D one's, these structure parameters can be designed so that both the separation of Pointing vectors and the difference in group velocities between orthogonal polarized modes

* Corresponding author. Tel.: +86 1062783389; fax: +86 1062770317.
E-mail address: xiaoli99@mails.thu.edu.cn (L. Xiao).

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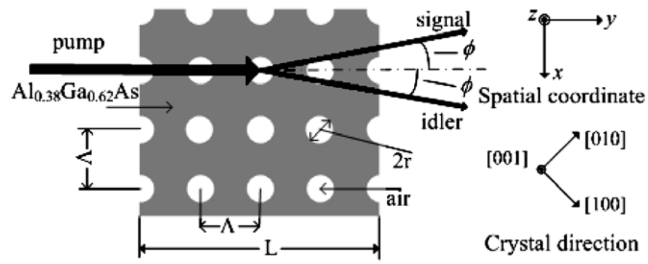

Fig. 1. Top view of the 2-D PC geometry with square lattices of $Al_{0.38}Ga_{0.62}As$/air.

can be minimized. Accordingly, a source of polarization entangled photon pairs with high nonlinear conversion efficiency and negligible walk-off could be realized.

The structure of the 2-D PC under investigation is an $Al_{0.38}Ga_{0.62}As$ plate with a square lattice of circular air holes, as shown in Fig. 1. Because AlGaAs crystal belongs to $4\bar{3}$ m point group, its quadratic susceptibility tensor has only one free $\boldsymbol{\chi}^{(2)}$ element $\chi_{14} = \chi_{25} = \chi_{36}$, while it is crucial to choose the lattice coordinates related to the crystal directions to make the nonlinear interaction efficient. The structure is designed by first considering an $Al_{0.38}Ga_{0.62}As$ plate without air holes, and setting the x and y axes of the lattice parallel to the [1 1 0] crystal directions, while z-axis is parallel to the [0 0 1] crystal direction. The plate is regarded to be infinitely thick (along z-axis) and $\mathbf{k}^z$ components of wave vectors are neglected. The TE mode (electric field parallel to the plate) and TM mode (electric filed perpendicular to the plate) are defined as two polarization states H and V, respectively. When the H monochromatic pump wave propagates along y-axis, the down-converted signal and idler lights with degenerate frequencies are emitted when phase-matching of the three waves is satisfied. The signal and idler waves propagate along the directions at the two sides of the pump light respectively in the x–y plane, as shown in Fig. 1. The choosing of crystal directions and the light transmission direction takes the best use of the quadratic susceptibility tensor. Calculation shows that only $H_p \rightarrow H_s + V_i$ and $H_p \rightarrow V_s + H_i$ processes are non-vanishing. If group velocities of H and V signal (or idler) lights are undistinguishable, only pure polarization entangled state $|\psi\rangle = (|V_iH_s\rangle + e^{i\alpha}|H_iV_s\rangle)/\sqrt{2}$ is generated in the SPDC process, wherein α is the relative phase shift which could be set as desired with additional wave plates. The corresponding effective nonlinear susceptibility is

$$
\begin{aligned}
\chi_{\text{eff}}^{\text{H}\rightarrow\text{H+V}} &= \chi_{\text{eff}}^{\text{H}\rightarrow\text{V+H}} = \hat{\mathbf{e}}_{\text{i}}^{\text{H}} \cdot \boldsymbol{\chi}^{(2)} : \hat{\mathbf{e}}_{\text{s}}^{\text{V}}\hat{\mathbf{e}}_{\text{p}}^{\text{H}} \\
&= \begin{bmatrix} \cos(\phi+\frac{\pi}{4}) \\ \cos(\phi-\frac{\pi}{4}) \\ 0 \end{bmatrix}^{\text{T}} \cdot \begin{bmatrix} 0 & 0 & 0 & \chi_{14} & 0 & 0 \\ 0 & 0 & 0 & 0 & \chi_{14} & 0 \\ 0 & 0 & 0 & 0 & 0 & \chi_{14} \end{bmatrix} \\
&: \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ \cos\frac{\pi}{4} \\ \cos\frac{\pi}{4} \end{bmatrix} = \cos\phi\,\chi_{14}
\end{aligned}
\tag{1}
$$

where ϕ is the angle between pump and signal or idler wave vectors. The quadratic nonlinear interaction Hamilton for the SPDC process is [11]

$$
\widehat{\text{H}}_{\text{int}} = \varepsilon_0 \int_{\text{V}} d\mathbf{r}\chi_{\text{eff}} E_{\text{p}}^*(\mathbf{r}) E_{\text{s}}(\mathbf{r}) E_{\text{i}}(\mathbf{r}) \tag{2}
$$

where $E_{\text{p,s,i}}$ are the complex amplitudes of electric field of pump, signal and idler waves, respectively. The integration covers all contributing volumes in the nonlinear medium.

In the 2-D PC shown in Fig. 1, $\chi_{\text{eff}}(\mathbf{r})$ is periodic in space and can be written as a Fourier expansion over the reciprocal lattice vectors $\mathbf{G}$ of 2-D PC as

$$
\chi_{\text{eff}}(\mathbf{r}) = \sum_{\mathbf{G}} \chi_{\text{eff}}(\mathbf{G}) e^{i\mathbf{G}\cdot\mathbf{r}} \tag{3}
$$

It is convenient to describe the propagation mode of electromagnetic wave in 2-D PC by a Bloch wave with frequency ω and wave vector $\mathbf{k}$ in the first Brillouin zone (BZ). A Bloch wave can be expressed as a Fourier sum of plane waves over all reciprocal lattice vectors $\mathbf{G}$:

$$
E_{\mathbf{k}}(\mathbf{r}, t) = e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \sum_{\mathbf{G}} \xi_{\mathbf{k}}(\mathbf{G}) e^{-i\mathbf{G}\cdot\mathbf{r}} \tag{4}
$$

where $\xi_{\mathbf{k}}(\mathbf{G})$ are the Fourier coefficients of the space harmonics. We consider the interaction of three forward propagating Bloch waves by substituting the Fourier expansions of $E_{\text{p,s,i}}(\mathbf{r})$ and $\chi_{\text{eff}}(\mathbf{r})$ into Eq. (2). The nonlinear interaction Hamilton is

$$
\begin{aligned}
\widehat{H}_{\text{int}} &= \varepsilon_0\delta_\omega \int d\mathbf{k}_{\text{s}}\, d\mathbf{k}_{\text{i}} \sum_{\mathbf{G}_\chi,\mathbf{G}_{\text{p}},\mathbf{G}_{\text{s}},\mathbf{G}_{\text{i}}} \chi_{\text{eff}}^{(2)}(\mathbf{G}_\chi)\xi_{\text{p}}^{(+)}(\mathbf{G}_{\text{p}})\xi_{\text{s}}^{(-)}(\mathbf{G}_{\text{s}})\xi_{\text{i}}^{(-)}(\mathbf{G}_{\text{i}}) \\
&\quad \times \int_{\text{V}} d\mathbf{r}\, e^{-i\Delta\mathbf{k}\cdot\mathbf{r}} + \text{h.c.} \\
&= \varepsilon_0\delta_\omega \int d\mathbf{k}_{\text{s}}\, d\mathbf{k}_{\text{i}} \sum_{\mathbf{G}_\chi,\mathbf{G}_{\text{p}},\mathbf{G}_{\text{s}},\mathbf{G}_{\text{i}}} \delta(\Delta\mathbf{k})\chi_{\text{eff}}^{(2)}(\mathbf{G}_\chi)\xi_{\text{p}}^{(+)}(\mathbf{G}_{\text{p}})\xi_{\text{s}}^{(-)} \\
&\quad \times (\mathbf{G}_{\text{s}})\xi_{\text{i}}^{(-)}(\mathbf{G}_{\text{i}}) + \text{h.c.}
\end{aligned}
\tag{5}
$$

where $\xi^{(+)}$ and its adjoint $\xi^{(-)}$ contain only annihilation and creation operators, respectively, $\Delta\mathbf{k} = \mathbf{k}_{\text{p}} + \mathbf{G}_{\text{p}} - \mathbf{k}_{\text{s}} - \mathbf{G}_{\text{s}} - \mathbf{k}_{\text{i}} - \mathbf{G}_{\text{i}} + \mathbf{G}_\chi$ is the quasi-phase-mismatching and h.c. stands for Hermitian conjugate, while $\delta_\omega = \delta(\omega_{\text{p}} - \omega_{\text{s}} - \omega_{\text{i}})$ relates to energy conservation.

The amplitudes of the Fourier components of a Bloch wave are related to each other as shown in (4), and one leading term holds most of the energy. Phase-matching ($\Delta\mathbf{k} = 0$) between the leading terms of pump, signal and idler Bloch waves is assumed to ensure the SPDC process efficient. The unmatched terms are neglected, which will reduce the nonlinear interaction efficiency to some degree. We consider the situation that the refraction indices of $Al_{0.38}Ga_{0.62}As$ are 3.38 for pump light at 775 nm and 3.17 for signal/idler light at 1550 nm [12]. Using the Plane Wave Method [13], the band structure of 2-D PC and dispersion surfaces at corresponding frequencies are calculated. For example, Fig. 2 is a sketch of the equi-frequency contour of the H pump light. Equi-frequency contours of the signal/idler lights are plotted at the origin and the

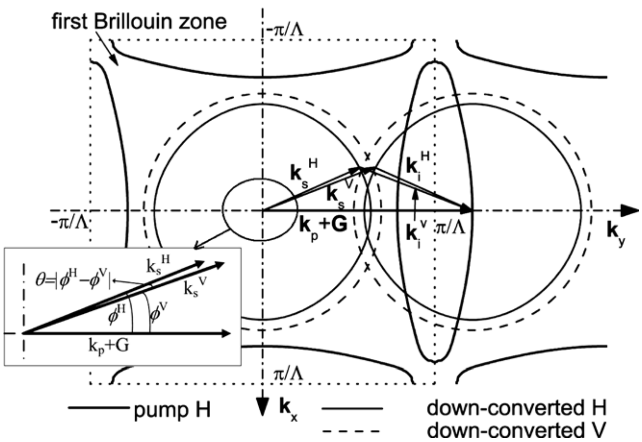

Fig. 2. Sketch of equi-frequency contours of the H pump wave and the down-converted H and V waves ($\Lambda = 200$ nm and $r = 65$ nm are selected for clarity). The magnified part shows the relation between the angles of the wave vectors.

end of the leading pump wave vector respectively. Intersections between the signal/idler equi-frequency contours correspond to phase-matching condition of the three leading wave vectors. The inset shows the separation between the orthogonal polarized photons, which is the cause of transverse walk-off. $\theta = |\phi^H - \phi^V|$ is the angle difference between the departure angles of down-converted H and V waves from the pump inside the PC.

Phase-matching and walk-off depend on the dispersion relation, which is sensitive to the 2-D PC structure. Fig. 3a shows the phase-matching condition versus Λ and r of the 2-D PC, which cannot be satisfied if the pump wave vector is located in the bandgap, or larger than the sum of wave vectors of the down-converted H and V waves. When it is phase-matched, the longitudinal and transverse walk-offs between the down-converted V and H lights for an interaction length L are defined as $\delta T = L|1/v_g^H - 1/v_g^V|$ and $d = L\tan\theta$, respectively, where v_g^H and v_g^V are the group velocities of the down-converted H and V lights. Fig. 3b shows the dependence of ϕ^H and ϕ^V on r under different Λ when phase-matching condition is satisfied. They are bigger in 2-D PC with larger air filling factor when the Λ keeps constant, and vice versa. It means the phase-matched ϕ can go up to much higher values in 2-D PC than in traditional method relying on material or structure birefringence. Fig. 3c shows the relation of walk-offs and r under different Λ when phase-matched. It indicates that within the range of parameters given, the longitudinal walk-off increases dramatically with Λ from the order of 10^{-2} ps to the order of ps, while the transverse walk-off keeps within the order of 10 μm. To minimize the walk-off, an appropriate design of structure parameters is needed to satisfy the phase-matching of leading terms of pump, signal/idler Bloch waves; and to make the group velocity difference between the down-converted H and V lights small enough. In addition, the structure parameter tolerance would be acceptable, since short interaction length can be expected for its high efficiency in such an application.

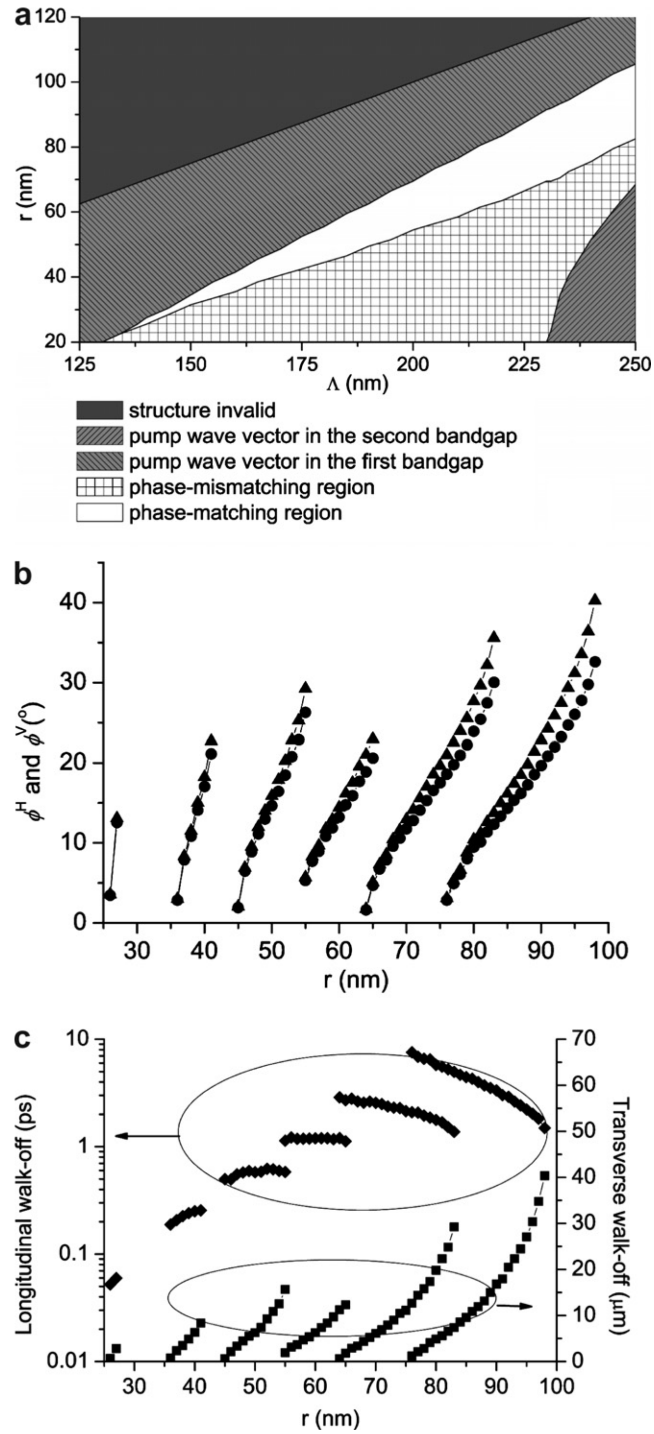

Fig. 3. (a) Phase-matching condition versus period Λ and hole radius r of 2-D PC. (b) ϕ^H(triangle) and ϕ^V (circle) versus r under different Λ. (c) Longitudinal (diamond) and transverse (square) walk-offs versus r under different Λ for a 2-D PC with $L = 3$ mm. Λ are equal to 140 nm, 160 nm, 180 nm, 200 nm, 220 nm, 240 nm for the lines from the left to the right in (b) and (c).

Taking $\Lambda = 138$ nm and $r = 25$ nm, the leading Fourier term of the down-converted H wave in the first BZ occupies 99.3% of the total energy, while the leading Fourier term of the down-converted V wave in the first BZ occupies 99.5% of the total energy, the leading Fourier term of H pump wave in the second BZ holds 71.8% of the total energy, respectively. The factor caused by Fourier expansion of χ_{eff} is 90% for $\mathbf{G}_\chi = 0$, equaling to the filling factor of $Al_{0.38}Ga_{0.62}As$ in the 2-D PC. Note that the quadratic nonlinear susceptibility of AlGaAs ($\chi_{14} \approx 343$ pm/V for $Al_{0.32}Ga_{0.68}As$ at 1550 nm and 134 pm/V for $Al_{0.8}Ga_{0.2}$) [14] is 2 orders of magnitude larger than that of BBO ($\chi_{22} \approx 2.2$ pm/V) [6]. The ratio of intensity down-conversion efficiency in the 2-D PC of $Al_{0.38}Ga_{0.62}As$/air to that in a BBO with the same length is $99.3\% \times 99.5\% \times 71.8\% \times (90\% \times \chi_{eff,2D\ PC}/\chi_{22,BBO})^2 \approx 10^3$. Wave vector departure angles of the down-converted H and V waves from the pump wave are $\phi^H = 6°7'32''$ and $\phi^V = 5°53'49''$, respectively. The group velocities are $0.3342c$ and $0.3341c$ for the down-converted H and V lights inside the 2-D PC, where c is the velocity of light in vacuum. For 2-D PC with $L = 3$ mm, the transverse walk-off is 12 μm, which is negligible compared to the coherent pump beam width (usually hundreds of micrometers). The longitudinal walk-off is 46 fs, which is much shorter than the coherence time (1.6 ps) which is determined by collection irises and interference filters (centered at 1550 nm, ~5 nm FWHM) [5]. Consequently, the associated labeling effect is minimized and the walk-off compensation of the polarization entangled photon pairs based on 2-D PC becomes unnecessary.

Above analysis is based on infinitely thick 2-D PC structure, however, only slabs with finite thickness can be realistically fabricated, which is guiding by total internal reflection in vertical direction. The slab thickness and the material for substrate and superstrate can modify the band structures, which may influence the numerical results given above. However, the effects of 2-D PC with proper period, which acts as strong modulated PC for pump light and weak modulated PC for signal/idle light, also can be expected: (1) Phase-matching of pump and signal/idle light can be achieved, since it has much stronger influence on the dispersion contours under pump wavelength than those under the down-converted wavelength; (2) the negligible walk-off also can be expected, for the small structure birefringence of the two orthogonal polarization states in weak modulated 2-D PC under the down-converted wavelength.

In conclusion, we have proposed a source of polarization entangled photon pairs with high nonlinear conversion efficiency and negligible walk-off, which is based on SPDC in a 2-D PC semiconductor with large quadratic nonlinearity. Although the Fourier expansion of Bloch waves and nonlinear susceptibility decreases the nonlinear interaction efficiency, it can be redeemed by the prominent large nonlinear susceptibility of the AlGaAs material. The overall SPDC efficiency is still higher than that in traditional crystals. Furthermore, 2-D PC has abnormal dispersion relation, which depends on both material dispersion and PC structure. Walk-off between the down-converted H and V lights can be designed to be small enough to minimize the labeling effect on polarization entangled photon pairs; hence the complicated walk-off compensation measures can be eliminated. The rapid progress in semiconductor technology for PC fabrication indicates the period of 2-D PC as small as 200 nm has been achieved nowadays, and the depth of the holes has reached several μms [15,16]. Therefore the 2-D PC could be an outstanding candidate for generation of polarization entangled photon pairs.

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