Efficient strategy for sharing entanglement via noisy channels with doubly entangled photon pairs

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Polarization-entangled photon pairs are easily perturbed in noisy channels. We propose an efficient strategy for sharing polarization-entangled photon pairs (PEPPs) using the additional frequency labels of polarization-frequency doubly entangled photon pairs (DEPPs). The DEPPs are used in transmission, followed by a two-step operation. In the first step, all the bit-flip noises are wiped off efficiently. In the second step, the frequency labels of the DEPPs are erased and the phase-flip noises are wiped off; thus PEPPs in the desired state are extracted. Theoretical analysis shows that this strategy has intrinsically high efficiency, which demonstrates that it has great potential in sharing entanglement via noisy transmission channels.

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The polarization-entangled photon pair (PEPP) is an important source in applications of quantum-information processing, such as quantum cryptography [1], quantum teleportation [2], quantum dense coding [3], and quantum error correction [4], because of its nonlocal correlation and advantages of easy generation and manipulation. Since the photon polarization is sensitive to perturbations in transmission channels, several purification schemes for PEPPs have been proposed [5,6]. However, the fidelity improvement is limited and an initial fidelity threshold is required in these schemes, which limits their feasibility in practice. High fidelity could be recovered by iterative purification [7] at the cost of intrinsically low purification efficiency, which is defined as the number ratio of the output pairs and input pairs, since at least half of the pairs should be sacrificed as targets in each iteration. Hence an efficient strategy for sharing PEPPs via noisy channels is still being sought.

Recently, entanglement with multiple degrees of freedom (DOFs) has attracted much attention. Ravaro *et al.* studied the feasibility of generating two-photon states which exhibit discrete frequency entanglement as well as polarization entanglement in semiconductor waveguides by bidirectional pumping and spontaneous parametric down-conversion [8]. Since operations in one DOF do not perturb the other DOF, such states may provide a new way to realize quantum-information functions. Aolita *et al.* showed that this method could be used in quantum communication without alignment [9].

In this paper, we propose a strategy for sharing PEPPs through noisy channels based on polarization-frequency doubly entangled photon pairs (DEPPs). In our strategy, the DEPPs are transmitted in quantum channels under polarization perturbation, and then distilled and converted to PEPPs by a two-step operation based on linear optical components. The efficiency of the distillation and conversion is high, thanks to the frequency entanglement property of the DEPPs.

Without losing generality, we suppose that the desired polarization-entangled state is

$$|\Phi^{+}\rangle = \frac{\sqrt{2}}{2}(|H\rangle|H\rangle + |V\rangle|V\rangle), \tag{1}$$

where $|H\rangle$ and $|V\rangle$ represent the horizontal and vertical polarizations, respectively. The DEPPs used for transmission are in the state

$$|\Phi_D^{+}\rangle = \frac{\sqrt{2}}{2}(|H,\omega_s\rangle|H,\omega_i\rangle + |V,\omega_s'\rangle|V,\omega_i'\rangle), \tag{2}$$

where s and i represent the signal and idler lights, respectively, according to the usage in nonlinear optics. ω_s and ω_s' are the possible frequencies of signal photons, while ω_i and ω_i' are those of idler photons. It can be seen that the state is entangled in both polarization and frequency. When DEPPs in a pure $|\Phi_D^{+}\rangle$ state are transmitted, the state will be transformed into a mixture composed of the following states:

$$|\Phi_D\rangle = \frac{1}{\sqrt{2}}(|H,\omega_s\rangle|H,\omega_i\rangle + e^{i\theta}|V,\omega_s'\rangle|V,\omega_i'\rangle),$$

$$|\Psi_D\rangle = \frac{1}{\sqrt{2}}(|H,\omega_s\rangle|V,\omega_i\rangle + e^{i\theta}|V,\omega_s'\rangle|H,\omega_i'\rangle),$$

$$|\Upsilon_D\rangle = \frac{1}{\sqrt{2}}(|V,\omega_s\rangle|H,\omega_i\rangle + e^{i\theta}|H,\omega_s'\rangle|V,\omega_i'\rangle),$$

$$|\Gamma_D\rangle = \frac{1}{\sqrt{2}}(|V,\omega_s\rangle|V,\omega_i\rangle + e^{i\theta}|H,\omega_s'\rangle|H,\omega_i'\rangle), \tag{3}$$

in which the θ's are arbitrary phases. $|\Phi_D\rangle$ includes the phase-flip noise, while $|\Psi_D\rangle$, $|\Upsilon_D\rangle$, and $|\Gamma_D\rangle$ are the bit-flip noises.

The first-step operation is depicted in Fig. 1. There is a polarization-independent wavelength division multiplexer (WDM) on each side of the DEPPs, which guides the photons to the right paths according to their frequencies. Then the photons are injected into a frequency-independent polarization beam splitter (PBS) on each side, which transmits the

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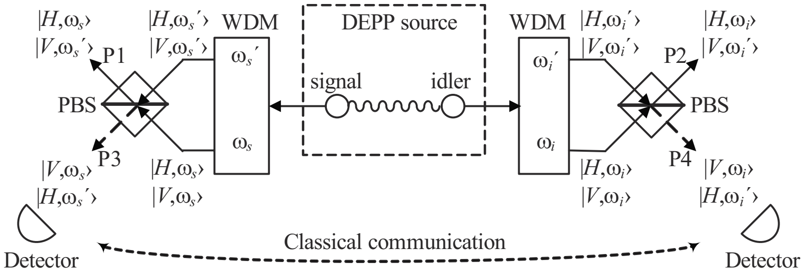

FIG. 1. Sketch of the first-step distillation for the perturbed DEPPs. The desired states are $|\Phi_D^{\pm}\rangle$ in (3), which are retained in $P1$ and $P2$.

photons in horizontal polarization and reflects the vertically polarized ones. On the signal side, photons in states $|H,\omega_s\rangle$ and $|V,\omega_s'\rangle$ reach $P1$, while photons in states $|V,\omega_s\rangle$ and $|H,\omega_s'\rangle$ reach $P3$. On the idler side, photons in states $|H,\omega_i\rangle$ and $|V,\omega_i'\rangle$ reach $P2$, while photons in states $|V,\omega_i\rangle$ and $|H,\omega_i'\rangle$ reach $P4$. The output ports for the reserved photon pairs are $P1$ and $P2$. If the input DEPP is in $|\Phi_D\rangle$, the pair is sent out from $P1$ and $P2$ without change. If it is in $|\Gamma_D\rangle$, the pair is sent out from $P3$ and $P4$, and discarded automatically. If it is in $|\Psi_D\rangle$ or $|\Upsilon_D\rangle$, one photon in the pair is sent out from the output port ($P1$ or $P2$), while the other is discarded ($P4$ or $P3$). Hence, it is transformed into a single-photon state. If two ideal detectors are located in $P3$ and $P4$, respectively, the pairs in the states of $|\Phi_D\rangle$ can be selected by saving the pairs when none of the detectors are triggered. As a result, all the bit-flip noises are eliminated and the fidelity is improved.

Using the frequency DOF of the DEPP as anadditional label to select the paths of photons with different polarizations, the first-step operation handles one DEPP at a time, without the need of a target pair; hence it is intrinsically efficient. Moreover, there is no threshold for the initial fidelity.

The DEPPs with phase-flip noise are further distilled and transformed into $|\Phi^+\rangle$ in the second step, which is depicted in Fig. 2. Suppose two neighboring pairs, namely, A and B, are exposed to the same phase perturbation [10]; they are both in $|\Phi_D\rangle$. First, the state of pair A is rotated to $|\Gamma_D\rangle$ by the half-wave plates (HWPs) on both sides. Then, the photons of the two pairs are injected into the PBS on each side from different directions. On the signal side, the photon with the frequency of ω_s, whose state is either $|H,\omega_s\rangle_B$ or $|V,\omega_s\rangle_A$, reaches $P1$, while the photon with the frequency of ω_s', whose state is either $|H,\omega_s'\rangle_A$ or $|V,\omega_s'\rangle_B$, reaches $P3$. Similarly, on the idler side, the photon in the state either $|H,\omega_i\rangle_B$ or $|V,\omega_i\rangle_A$ reaches $P2$, while the photon in either $|H,\omega_i'\rangle_A$ or $|V,\omega_i'\rangle_B$ reaches $P4$. Since each port is for photons with a certain frequency, in the following analysis, the photon frequencies are replaced by the port numbers for clarity. The whole four-photon state can be expressed as

$$|\Pi\rangle = |\Phi_D\rangle_B \otimes |\Gamma_D\rangle_A = \frac{1}{2}(e^{i\theta}|\Pi_1\rangle + |\Pi_2\rangle + e^{i2\theta}|\Pi_3\rangle), \quad (4)$$

where

$$|\Pi_1\rangle = |H,P1\rangle_B|H,P2\rangle_B|H,P3\rangle_A|H,P4\rangle_A + |V,P1\rangle_A|V,P2\rangle_A|V,P3\rangle_B|V,P4\rangle_B,$$

$$|\Pi_2\rangle = |H,P1\rangle_B|H,P2\rangle_B|V,P1\rangle_A|V,P2\rangle_A,$$

$$|\Pi_3\rangle = |V,P3\rangle_B|V,P4\rangle_B|H,P3\rangle_A|H,P4\rangle_A. \quad (5)$$

The photons sent out from $P1$ and $P2$ are saved, while the photons sent out from $P3$ and $P4$ are locally projected onto the diagonal bases, which are expressed as $|D^{\pm}\rangle = (|H\rangle \pm |V\rangle)/\sqrt{2}$, by a HWP and a PBS in each port, and then detected. If the four-photon state is in $|\Pi_2\rangle$, all the photons are sent out from $P1$ and $P2$, which can be discarded artificially as no click is recorded by detectors on either side. If the four-photon state is in $|\Pi_3\rangle$, both detectors in $P3$ and $P4$ click twice, showing that there is no photon sent out from $P1$ or $P2$. Hence only the four-port state $|\Pi_1\rangle$ needs to be considered, in which only one photon is sent out from each port. $|\Pi_1\rangle$ can be further expressed as

$$|\Pi_1\rangle = |\Phi^+\rangle_{1,2} \otimes |\Phi^+\rangle_{3,4} + |\Phi^-\rangle_{1,2} \otimes |\Phi^-\rangle_{3,4}, \quad (6)$$

where

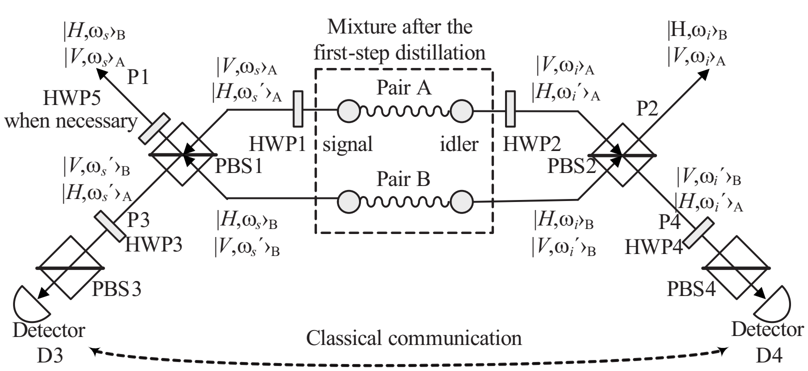

FIG. 2. Sketch of the second-step operation. The desired state is $|\Phi^+\rangle$ in (1), which is sent out from $P1$ and $P2$. HWP1 and HWP2 are two half-wave plates that change the state of pair A from $|\Phi_D\rangle$ to $|\Gamma_D\rangle$. HWP3, PBS3, HWP4, and PBS4 are half-wave plates and polarization beam splitters projecting the state of the photons in $P3$ and $P4$ onto the diagonal bases, in order to distinguish $|\Phi^+\rangle_{3,4}$ and $|\Phi^-\rangle_{3,4}$. HWP5 in $P1$ is a half-wave plate changing the state of the final PEPPs from $|\Phi^-\rangle$ to $|\Phi^+\rangle$ if necessary.

$$|\Phi^{\pm}\rangle_{1,2} = \frac{\sqrt{2}}{2}(|H,P1\rangle|H,P2\rangle \pm |V,P1\rangle|V,P2\rangle),$$
$$|\Phi^{\pm}\rangle_{3,4} = \frac{\sqrt{2}}{2}(|H,P3\rangle|H,P4\rangle \pm |V,P3\rangle|V,P4\rangle). \qquad (7)$$

$|\Phi^{\pm}\rangle_{1,2}$ are states of PEPPs sent out from $P1$ and $P2$, while $|\Phi^{\pm}\rangle_{3,4}$ are states of PEPPs sent out from $P3$ and $P4$. Equation (6) shows that, if the result of the combined detection in $P3$ and $P4$ is correlated, i.e., both photons in $|D^{+}\rangle$ or both in $|D^{-}\rangle$, the PEPP in $P1$ and $P2$ is in the $|\Phi^{+}\rangle$ state. On the contrary, if the combined detection result in $P3$ and $P4$ is anticorrelated, i.e., one photon in $|D^{+}\rangle$ and the other in $|D^{-}\rangle$, the PEPP in $P1$ and $P2$ is in the $|\Phi^{-}\rangle$ state, which can be converted to $|\Phi^{+}\rangle$ by a HWP in $P1$. As a result, the frequency labels are erased and the PEPPs in the desired pure state $|\Phi^{+}\rangle$ are obtained in $P1$ and $P2$.

On the assumption that the two neighboring pairs are exposed to the same perturbation, the fidelity of the output state is 100% after the second-step operation. The efficiency of this step is 25%, since half pairs are sacrificed as targets and the probability of the four-photon state in $|\Pi_1\rangle$ in (5) is 50%. However, the neighboring pairs may not always be in the same state. The probability of the neighboring pairs being identical can be measured using a fraction of the pairs, which is defined as the parameter g. The final fidelity of $|\Phi^{+}\rangle$ equals g. We define the ratio of the dominating state as the initial fidelity of this step operation, which is represented by F_0. If $g > F_0$, the fidelity is improved since the ratio of $|\Phi^{+}\rangle$ after the operation of this step equals g. Otherwise, the effect of the second-step operation is only to erase the frequency labels. The conventional iterative purification [7] can be used for further fidelity improvement. In any case, in a real environment, the rate of perturbations due to the environmental variations is always lower than the bit rate. Hence, high fidelity could be recovered since g is always close to 1 no matter what is the value of F_0.

The strategy is feasible for any mixture composed of the states in Eq. (3). To show the performance of the strategy in realistic perturbed channels, two specific examples are analyzed, in which DEPPs in the pure $|\Phi_D^{+}\rangle$ state are used in transmission. The state is changed into a mixture by perturbations, and then distilled and converted to $|\Phi^{+}\rangle$ using the two-step operation. In the analysis, we focus on the fidelities and efficiencies of the operation. The parameter g is taken as 1, supposing that the bit rate of DEPPs is much faster than the perturbations due to the slow environmental variation. This means that the fidelity of the output state is 100%. The performance of the conventional iterative purification on PEPPs exposed to the same perturbation is also calculated for comparison. The perturbations in the noisy channels can be simulated by Pauli rotations, which are denoted by $\sigma_{x,y,z}$ for rotation of π rad about the x,y,z axes, respectively. The complementary bases of the doubly entangled states include

$$|\Phi_D^{\pm}\rangle = \frac{1}{\sqrt{2}}(|H,\omega_s\rangle|H,\omega_i\rangle \pm |V,\omega_s'\rangle|V,\omega_i'\rangle),$$
$$|\Psi_D^{\pm}\rangle = \frac{1}{\sqrt{2}}(|H,\omega_s\rangle|V,\omega_i\rangle \pm |V,\omega_s'\rangle|H,\omega_i'\rangle),$$
$$|\Upsilon_D^{\pm}\rangle = \frac{1}{\sqrt{2}}(|V,\omega_s\rangle|H,\omega_i\rangle \pm |H,\omega_s'\rangle|V,\omega_i'\rangle),$$
$$|\Gamma_D^{\pm}\rangle = \frac{1}{\sqrt{2}}(|V,\omega_s\rangle|V,\omega_i\rangle \pm |H,\omega_s'\rangle|H,\omega_i'\rangle), \qquad (8)$$

in which $|\Phi_D^{-}\rangle$ is transformed from $|\Phi_D^{+}\rangle$ by unilateral σ_z rotation, while $|\Psi_D^{\mp}\rangle$, $|\Upsilon_D^{\mp}\rangle$, and $|\Gamma_D^{\pm}\rangle$ are transformed from $|\Phi_D^{\pm}\rangle$ by σ_x rotations on the idler photon, signal photon, and both of them, respectively.

In the first example, the perturbation appears only in a single side of the quantum channel, for example, the idler side. Unilateral perturbations, which are simulated by the three Pauli matrices each with a probability of p, change the pure $|\Phi_D^{+}\rangle$ state into

$$W = F|\Phi_D^{+}\rangle\langle\Phi_D^{+}| + \frac{1-F}{3}|\Phi_D^{-}\rangle\langle\Phi_D^{-}| + \frac{1-F}{3}|\Psi_D^{+}\rangle\langle\Psi_D^{+}| + \frac{1-F}{3}|\Psi_D^{-}\rangle\langle\Psi_D^{-}|, \qquad (9)$$

where $F=1-3p$ is the fidelity of $|\Phi_D^{+}\rangle$, defined as the ratio of DEPPs in $|\Phi_D^{+}\rangle$. According to the above description, the fidelity of $|\Phi_D^{+}\rangle$ in the mixture after the first-step operation is $F'=(1+2F)/3$. If the PEPPs in the pure state $|\Phi^{+}\rangle$ are transmitted and exposed to the same unilateral perturbations, it is changed into a mixture in the same form as (9), in which $|\Phi_D^{\pm}\rangle$ and $|\Psi_D^{\pm}\rangle$ should be replaced by $|\Phi^{\pm}\rangle$ and $|\Psi^{\pm}\rangle$. The fidelity of $|\Phi^{+}\rangle$ after perturbation is denoted by F_p, and the fidelities after the first and second iterations of the purification scheme in Ref. [7] are denoted by $F'_{p,1}$ and $F'_{p,2}$, respectively. Figure 3(a) shows the fidelities of F, F', F_p, $F'_{p,1}$, and $F'_{p,2}$ under different unilateral perturbation probabilities. F_p is the same as F, while $F'_{p,1}$ and even $F'_{p,2}$ are much lower than F' in most cases. The fidelity improvement ratios of the first-step purification in both the DEPP and PEPP cases are plotted in Fig. 3(b). It can be seen that F'/F for our strategy is always larger than 1, which means that there is no threshold for the initial fidelity, while $F'_{p,1}/F_p$ for the conventional purification is larger than 1 only if $F_p > 1/2$.

In our strategy, the first two terms in (9) remain after the first-step operation, with an efficiency of $\eta_1=(1+2F)/3$. Then the mixture can be further distilled and converted to $|\Phi^{+}\rangle$ by the second-step operation with an efficiency of 25%. Hence, the total efficiency of our strategy is $\eta_2=(1+2F)/12=(1-2p)/4$. Figure 3(c) shows the efficiencies of η_1 and η_2 under different unilateral perturbation probabilities. As a comparison, the efficiencies of the iterative purification scheme for PEPPs after the first and second iterations are also plotted in Fig. 3(c), denoted by $\eta_{p,1}$ and $\eta_{p,2}$, respectively. It can be seen that η_1 is higher than $\eta_{p,1}$, while the total efficiency of our strategy η_2 is higher than the efficiency of the iterative purification of PEPPs with the number of iterations larger than 2.

The more general case is that the DEPPs are exposed to bilateral perturbations, which are simulated by the Pauli matrices operating on both sides, each with a probability of p. The mixture after perturbation can be expressed as

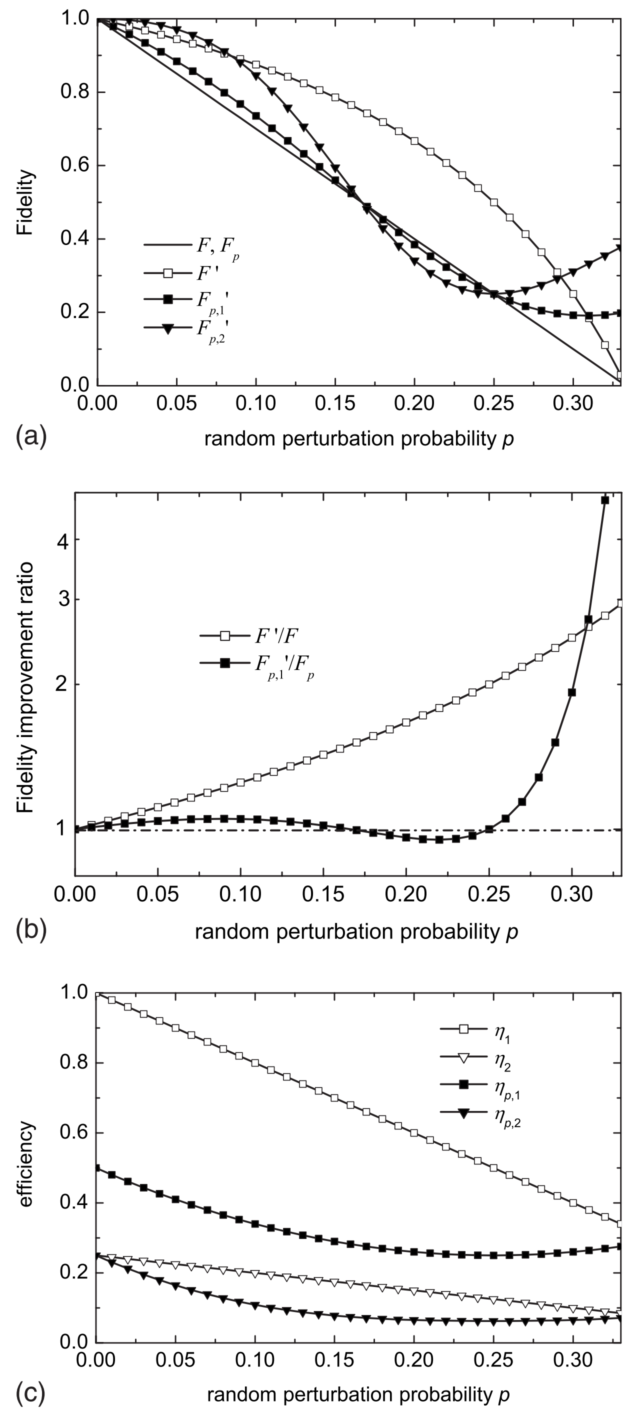

FIG. 3. Performance comparison between the proposed strategy and the iterative PEPP purification scheme under different unilateral perturbation probabilities. (a) The fidelities versus unilateral perturbation probability. (b) The fidelity improvement ratios of the first-step operation and the conventional PEPP purification operated once. (c) The efficiencies versus unilateral perturbation probability.

$$
\begin{aligned}
W_d = F|\Phi_D^+\rangle\langle\Phi_D^+| + R_1|\Phi_D^-\rangle\langle\Phi_D^-| \\
+ R_2|\Psi_D^+\rangle\langle\Psi_D^+| + R_2|\Psi_D^-\rangle\langle\Psi_D^-| \\
+ R_2|\Upsilon_D^+\rangle\langle\Upsilon_D^+| + R_2|\Upsilon_D^-\rangle\langle\Upsilon_D^-| \\
+ R_3|\Gamma_D^+\rangle\langle\Gamma_D^+| + R_3|\Gamma_D^-\rangle\langle\Gamma_D^-|,
\end{aligned}
$$

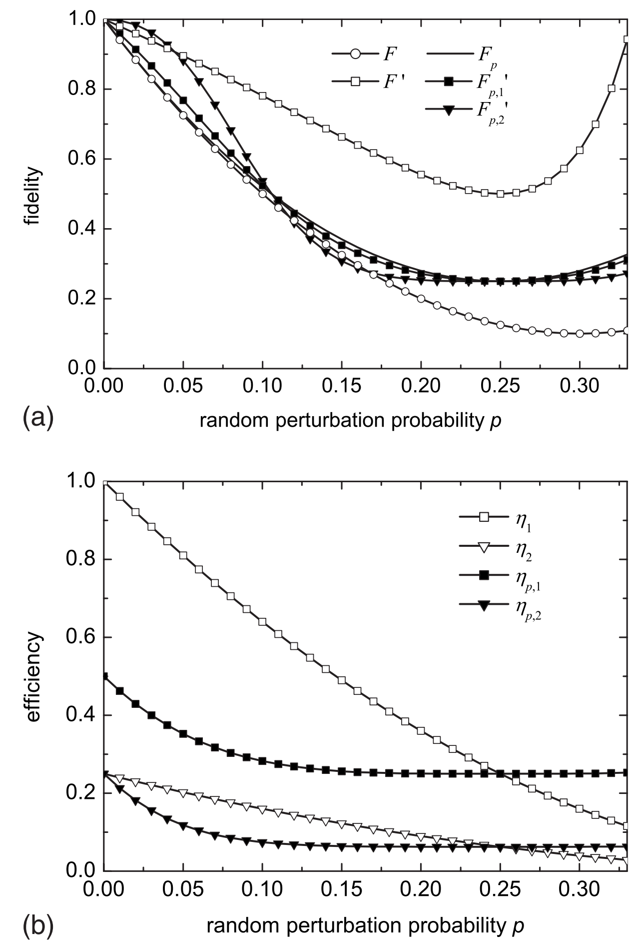

FIG. 4. Performance comparison between the proposed strategy and the iterative PEPP purification scheme under different bilateral perturbation probabilities. (a) The fidelities versus unilateral perturbation probability. (b) The efficiencies versus unilateral perturbation probability.

$$
\begin{aligned}
F = 10p^2 - 6p + 1, \quad R_1 = 2p - 6p^2, \\
R_2 = p - 2p^2, \quad R_3 = 2p^2. \qquad (10)
\end{aligned}
$$

The fidelity of $|\Phi_D^+\rangle$ after perturbation is denoted by F in (10). After the first-step operation of our strategy, the fidelity of $|\Phi_D^+\rangle$ is improved to $F' = F/(F+R_1)$. In the case of PEPP transmission, the pure $|\Phi^+\rangle$ state is changed into a mixture in the same form as (9), in which $|\Phi_D^\pm\rangle$ and $|\Psi_D^\pm\rangle$ should be replaced by $|\Phi^\pm\rangle$ and $|\Psi^\pm\rangle$. The fidelity of $|\Phi^+\rangle$ is $F_p = 12p^2 - 6p + 1$. The fidelities after the first and second iterations of the purification scheme in Ref. [7] are denoted by $F_{p,1}'$ and $F_{p,2}'$, respectively. Figure 4(a) shows the fidelities F, F', F_p, $F_{p,1}'$, and $F_{p,2}'$ under different bilateral perturbation probabilities. Although F for DEPPs is lower than F_p for PEPPs, after the first-step operation, F' is much higher than $F_{p,1}'$ and $F_{p,2}'$ in most cases.

As in the unilateral perturbation case, only the first two terms in (10) are left after the first-step operation, with an efficiency of $\eta_1 = F + R_1 = (1-2p)^2$. Then the mixture can be further fully distilled and converted to $|\Phi^+\rangle$ by the second-step operation, with a total efficiency of $\eta_2 = (1-2p)^2/4$. Figure 4(b) shows the efficiencies of η_1 and η_2 under different bilateral perturbation probabilities. As a comparison, the efficiencies of the iterative purification for PEPPs after the first and second iterations are also plotted.

The above analysis is based on the assumption that the whole system is lossless and the detectors are perfect. However, such nonideal conditions would impact the performance of our strategy in practice. According to the feasibility analysis of the conventional PEPP purification schemes [11], the impacts of channel loss and nonideal detection are different in different application environments. In the post-selected environment in which coincidence detection is necessary, the vacuum state and the single photons, as well as the nonideal detection, would not affect the fidelity of the entangled states after our operation. However, the efficiency of the strategy is much lower than expected. In the non-post-selected environment, the effective fidelity should be defined as the ratio of the desired entangled state to the whole output mixture including the entangled states, single-photon states, and the vacuum state, which is highly dependent on the detection efficiency. Our first-step operation may increase the ratio of the single-photon states and the vacuum state to a certain degree, considering the nonideal detection. The impact of limited detection efficiency on the second-step operation, in which the four-port state is reserved according to the detection results, is similar to the analysis in Ref. [10].

In conclusion, we have shown that with DEPPs and the two-step operation, a polarization-entangled state with high fidelity can be achieved after transmission and perturbation. In the first-step operation, the bit-flip noises in the perturbed DEPPs are wiped off using the additional frequency labels. The method is efficient since it handles one pair at a time without the need of target pairs. In the second step, the frequency labels of the output DEPPs of the first-step operation are erased and the PEPPs in the desired state are extracted out, at a cost of half the pairs being sacrificed. The efficiency of the whole process is higher than that of the iterative purification scheme of PEPPs, which means that more qualified pairs can be obtained after transmission and distillation. Hence this strategy has many potential applications in sharing entanglement via noisy transmission channels.

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