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# Abnormal Cutoff Thickness of Long-Range Surface Plasmon Polariton Modes Guided by Thin Metal Films *

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*Long-range surface plasmon polariton (LRSPP) modes guided by a thin metal film surrounded by semi-infinite dielectrics with different refractive indices are studied. Our calculation results show that the cutoff thickness of the metal film does not monotonically increase with refractive index difference $\Delta n$ between the substrate and superstrate. Just because of this abnormal behaviour of cutoff thickness, the existence of LRSPP illustrates complicated situations in asymmetric configurations. For a certain metal film thickness, LRSPP may exist in one, two or three refractive index difference $\Delta n$ regions.*

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Surface plasmon polariton (SPP) is transverse-magnetic surface electromagnetic excitation that propagates in a wave-like fashion along the interface between a metal and a dielectric medium, and whose amplitude decays exponentially with increasing distance into each medium from the interface.[1,2] For a thin metal film surrounded by two semi-infinite dielectrics, the SPPs on the two interfaces are coupled and form four types of different modes. One of them called the long-range surface plasmon polariton (LRSPP) attracts much attention because its transmission loss decreases with the film thickness[3,4,5] and it can be excited easily and efficiently.[4,6,7] LRSPP-based integrated devices and circuits are being increasingly used for light routing and switching in the rapidly developing area of broadband optical communications.[8−10] Meanwhile, the high sensitivity to the surrounding circumstances of LRSPP leads to controllable devices[11,12] and SPP sensors.[13,14]

LRSPP modes supported by thin metal films and metal strips surrounded by semi-infinite dielectrics with different refractive indices, namely asymmetric structure, are promising for various applications because they are easier to fabricate and more flexible in use. The controllable device based on LRSPP[11,12] can be more compactable than the conventional prism structure.[15] However, to support LRSPP for larger refractive index difference $\Delta n$, a thicker metal film, which leads to larger loss, is needed. This is because the cutoff thickness of LRSPP increases monotonically with $\Delta n$. Our calculation result shows a different situation that cutoff thickness of metal film does not increase monotonically with $\Delta n$. Thus, with large $\Delta n$, a thin metal film can be used to support LRSPP.

LRSPP supported by a thin metal film and its characteristics have been studied in symmetric and asymmetric structures for more than twenty years. In recent years, Berini[16] showed that metal strip supported SPP modes have complex dispersion curves. Although LRSPP has been studied for many years, researchers can just conclude not only for metal film but also for metal strip that cutoff thickness increases monotonically with $\Delta n$. This directly results in the existence of LRSPP in one $\Delta n$ region for fixed metal film thickness. In this Letter, a different situation is presented. It is found that the imaginary part of complex propagation constant of LRSPP mode, which decides the loss and existence of LRSPP mode, exhibits a complicated situation, when varying both the metal film thickness and the refractive index difference $\Delta n$ in a large region. Our calculation shows that for different metal film thicknesses, LRSPP may exist in one, two or three refractive index difference $\Delta n$ regions. The occurrence of the above-mentioned complicated situations is because the cutoff thickness of metal film does not increase monotonically with refractive index difference $\Delta n$.

The geometry under discussion is that of a metal film surrounded by two semi-infinite dielectrics. The substrate dielectric with refractive index $n_1$ fills the $z < 0$ region, the metal layer $0 < z < h$ is characterized by frequency-dependent dielectric function $\varepsilon_m(\omega)$, and the superstrate dielectric with refractive index $n_3$ fills the $z > h$ region. We assume that the LRSPP waves propagate in the $x$ direction with the complex propagation constant $\beta$ and the field in the $y$ direction is uniform. There are four types of different modes due to coupling of the SPPs on the two interfaces of the thin metal film. One of them is symmetric bound mode (SB), i.e. the so-called LRSPP. According to Maxwell's equations and electromagnetic field consistence on the interfaces, we can obtain the dis-

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persion relation[4]

$$\tanh(S_2h)(\varepsilon_1\varepsilon_3S_2^2+\varepsilon_m^2S_1S_3) + [S_2(\varepsilon_1S_3+\varepsilon_3S_1)\varepsilon_m]=0, \quad (1)$$

with

$$S_2^2=\beta^2-\varepsilon_{\rm m}k_0^2,\quad S_j^2=\beta^2-\varepsilon_jk_0^2\ (j=1,3). \quad (2)$$

Other three modes, antisymmetric bound (AB), antisymmetric leaky (AL), symmetric leaky (SL), can also be derived by solving the above dispersion equation. Nevertheless, this study merely focuses on the LRSPP, SB mode. Because this mode can bound the field on the metal interfaces and has a long propagation length, LRSPP has attracted much interesting and is widely investigated. In the calculation, we choose Al and Au with different substrates.

For Al ($\varepsilon_m=-29.8+i11.6$) at $\lambda_0=633$ nm, fixing the substrate refractive index $n_1=1.462$ and varying the superstrate refractive index $n_3$ from 1.0 to 5.0, the absolute value of attenuation constant $|\beta_I/k_0|$ of LRSPP, which corresponds to the propagation length ($L_{sp}=1/(2\beta_I)$),[2] was calculated and shown in Fig. 1. Here the $x$ axis just shows from 1.0 to 3.0 to illustrate the curve more clearly near 1.462. The flatness of the curves between 3.0 and 5.0 is because they are far from the cutoff points. Figure 1(a), 1(b), 1(c), and 1(d) correspond to four different metal film thickness of $h=18, 16, 12$, and 22 nm, respectively. There are two reasons for only illustrating the imaginary part of propagation constant $\beta_I$ here. First, only $\beta_I$ can tell us the loss (propagation length) of the mode and whether LRSPP mode exists or not. Second, the trend and the relation of the real part of propagation constant $\beta_R$ between the four modes are independent of the metal film thickness and almost the same as that shown by Zervas.[17]

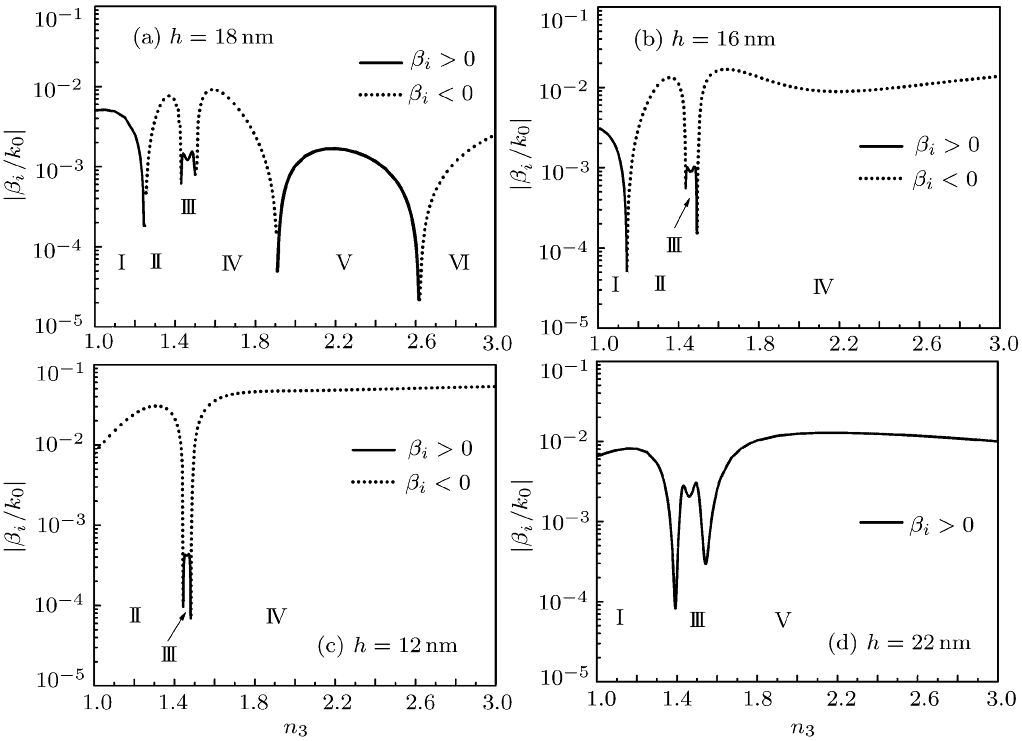

**Fig. 1.** Absolute value of attenuation constant ($|\beta_I/k_0|$) of the LRSPP modes versus the superstrate refractive index $n_3$ for different Al film thickness: (a) $h=18$ nm, (b) $h=16$ nm, (c) $h=12$ nm, (d) $h=22$ nm. Here $n_1=1.462$, $\varepsilon_m=-29.8+i11.6$, and $\lambda_0=633$ nm.

As depicted in Fig. 1, for different metal film thickness $h$, the differences between these four curves are obvious. The solid lines with $\beta_I>0$ indicate the existence of LRSPP and the SPP fields are bounded on the interfaces of the metal film. While the dotted lines correspond to $\beta_I<0$ and there exist no bound mode.[4] In Fig. 1(a), superstrate index range is divided into six regions (I, II, III, IV, V and VI) by successive cutoff points. LRSPP can only exist in region I, III and V (with $\beta_I>0$), while in region II, IV and VI (with $\beta_I<0$), the mode is a growing type[4] with no LRSPP existing. To our knowledge, such a complex situation is not reported before. Figure 1(b) shows that there are four regions (I, II, III and IV) left when $h=16$ nm and LRSPP can only exist in regions I and III.[17] What should be emphasized here is that this strongly asymmetric configuration when $h=16$ nm has a lower superstrate refractive index compared to the fixed substrate refractive index. If the refractive index of the superstrate is larger than that of the substrate, no LRSPP can be guided. This is different from Fig. 1(a), where LRSPP can exist on both sides (with a lower or larger superstrate refractive index).

Further thin the metal film h to 12 nm, regions I, V and Ⅵ disappear (Fig. 1(c)) and LRSPP can only exist in quasi-symmetric circumstance. That means that there is a maximum $\Delta n$ to support LRSPP, which has been reported by Yang *et al.*[5] with the approximations of the thin film and small $\Delta n$. In Fig. 1(d), where $h$ has a larger value of 22 nm, regions Ⅱ, Ⅳ and Ⅵ disappear and the superstrate refractive index does not affect the existence of LRSPP. The difference of the results in Fig. 1 with that in Ref. [17] results from changing the film thickness $h$ in dispersion relation (1). Especially for Figs. 1(a) and 1(b), film thickness 2 nm is easy to be ignored.

From the calculation results shown in Fig. 1, we have seen that the existence of LRSPP shows a complex situation in asymmetric configurations. To further understand how the metal film thickness $h$ and dielectric refractive index affect the existence of LRSPP, the cutoff points between the odd ( $\beta_I > 0$) and even ($\beta_I < 0$) regions of superstrate refractive index with fixed substrate are calculated as a function of $h$. They leads to the above complex dispersion relationship. The results are plotted in Fig. 2. To reveal the details more clearly in the low $n_3$ region, a logarithmic $y$ coordinate is adopted.

In the figure, the light grey (region Ⅱ) and dark grey (regions Ⅳ and Ⅵ) areas correspond to superstrate refractive index $n_3$ regions where no LRSPP can be supported ($\beta_I < 0$). For $n_1 = 1.462$ (Fig. 2(a)), when $h < 14.27$ nm, LRSPP can only exist in quasi-symmetric region (Ⅲ), corresponding to the case of Fig. 1(c). When $14.27 < h < 17.6$ nm, we can derive a similar conclusion mentioned in Ref. [17] that LRSPP can be supported in two regions (I and Ⅲ). In the case of $h > 17.6$ nm, region V, which divides the dark grey region into Ⅳ and Ⅵ, appears and its width grows rapidly when $h$ increases. Figure 1(a) reveals the most complicated situation, in the range of $17.6 < h < 21.84$ nm, where LRSPP can exist in regions I, Ⅲ, V. For comparative thick films ($h > 21.95$ nm), shown in Fig. 2(a), the existence of LRSPP is independent of $n_3$ within our concerned refractive index region. Careful readers may have noticed that with the increase of $h$, even regions (Ⅱ, Ⅳ and Ⅵ with $\beta_I < 0$) are narrowed and region Ⅳ disappears earlier than Ⅱ, which is more obvious in Fig. 2(b), where in a very small range of $h$ (less than 0.5 nm), just one of the six regions, region Ⅳ, disappears and region Ⅱ is very narrow. This situation is not shown in Fig. 1. Notice that in Fig. 2, the situations are simple for very thin and comparative thick metal films, and the complicated situations merely occur within an 8 nm film thickness range, from 14 nm to 22 nm, in the case of $n_1 = 1.462$ (Fig. 2(a)). For a larger substrate refractive index $n_1 = 2.0$ (Fig. 2(b)), this range is narrowed (just 6 nm from about 15.5 to 21.5 nm). The insets of Fig. 2(a) show the amplitude of magnetic field corresponding to I, Ⅲ, V regions. In region Ⅲ, the field is nearly symmetric, while in regions I and V, field decays slower in the higher refractive index side and do not as symmetric as that in region Ⅲ.

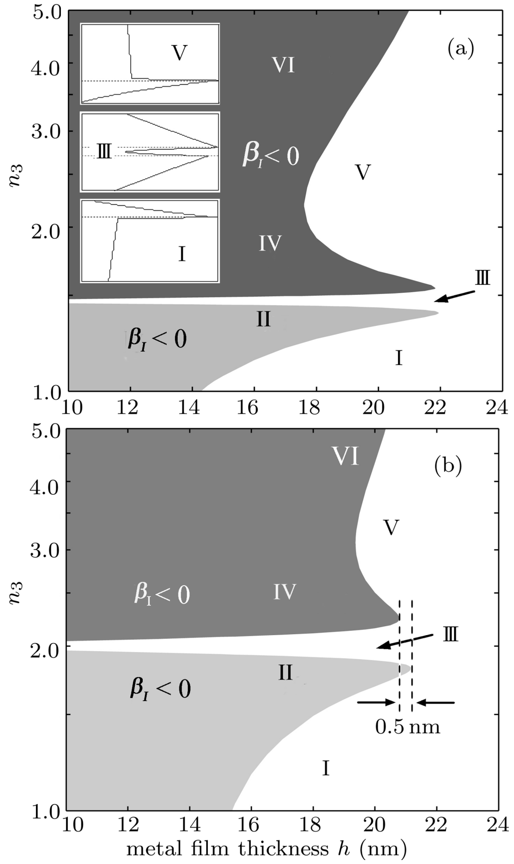

**Fig. 2.** Cutoff points between odd ($\beta_I > 0$) and even ($\beta_I < 0$) regions of superstrate refractive index as a function of $h$ with fixed substrate: (a) $n_1 = 1.462$, (b)$n_1 = 2.0$.

As depicted in Fig. 2, the interfaces of white and grey region are the cutoff point of LRSPP mode and correspond to the deeps of the curve in Fig. 1. In Refs. [4,16], $h_{co}$ increases monotonically with refractive index difference $\Delta n$, and thicker metal film has to be used to support LRSPP with large $\Delta n$. Thicker metal film results in larger LRSPP loss.[4] It is significant that our calculation shows that $h_{co}$ does not increase monotonically with $\Delta n$. It is this abnormal situation that results in LRSPP mode existing in one, two or three n regions for certain metal film thicknesses, and makes it possible that in the case of large $\Delta n$, thinner meal film can also support LRSPP mode.

Compared with Al, we find that Au and Ag have much lower imaginary part of dielectric constant. Thus the propagation loss $\beta_I$ of LRSPP should be much smaller. Taking Au ($\varepsilon_m = -132 + i12.65$) at $\lambda_0 = 1550\,\text{nm}$[18] with $n_1 = 1.6$ for example, it is found that similar situation of $\beta_I/k_0$ is derived as shown in Fig. 3. Although the thickness of the Au film in Fig. 3 is larger than that of Al, the propagation constant is much lower. The different situations in Fig. 3 also result from the abnormal cutoff thickness.

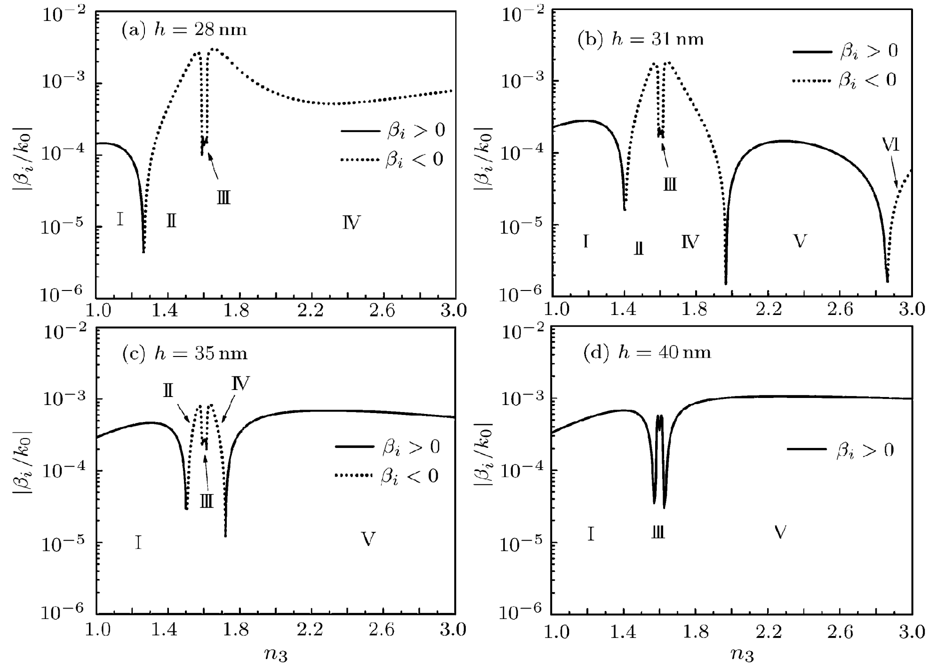

**Fig. 3.** Normalized imaginary part of the complex propagation constant ($|\beta_I/k_0|$) of the LRSPP modes versus the superstrate refractive index $n_3$ for different Au film thickness: (a) $h = 28\,\text{nm}$, (b) $h = 31\,\text{nm}$ , (c) $h = 35\,\text{nm}$, (d) $h = 40$ nm. Here $n_1 = 1.6$, $\varepsilon_m = -132 + i12.65$, and $\lambda_0 = 1550\,\text{nm}$.

In conclusion, the existence condition of LRSPP that is supported by a metal film surrounded by asymmetric semi-infinite dielectrics has been investigated. We find that a complicated existence situation of LRSPP in asymmetric configurations. For a very thin film, LRSPP can only exist within a small refractive index difference $\Delta n$ between the surrounding dielectrics. For a comparative thick film, the existence of LRSPP is almost independent of $\Delta n$. However, in a small range of film thickness, the situation becomes complicated, where LRSPP can exist in two or three $\Delta n$ regions. The occurrence of these complicated situations is due to the fact that the cutoff thickness of metal film does not increase monotonically with refractive index difference $\Delta n$. We expect that LRSPP with large refractive index difference can not only be used in the sensor field but also in the integrated optical devices with easier and more flexible fabrication technology.

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